MOTION OF SUSPENSIONS IN NARROW HORIZONTAL CHANNELS

I. F. Belova, Yu. V. Zheltov, and Yu. P. Zheltov

Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 136-140, 1965

The problem of the motion of coarse suspensions in horizontal cracks (narrow channels) arose in connection with the development of the technology of hydraulic fracturing of oil-bearing formations [1-3]. In using this method it is necessary to pump into cracks formed in the rocks a mixture of a liquid and a granular solid which serves as a consolidating agent. The granular material is usually a coarse-grained (0.6-0.8 mm) sand, while the liquid phase is a viscous liquid (oil, mazout, diesel fuel, sulfite liquor, etc.).

The steady motion of such suspensions in narrow channels with impermeable walls was studied in [4, 6]. The present article is devoted to a study of the unsteady motion of these mixtures in horizontal cracks with impermeable walls.

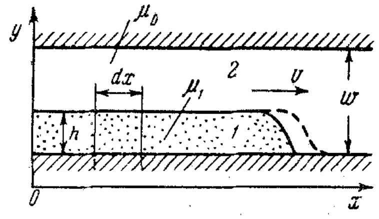

Fig. 1

The method described, based on the analogy [7] between the mechanisms of percolation of immiscible liquids in a porous medium and the motion of a mixture of sand and liquid in a narrow channel, makes it possible to determine the basic technical parameters of the process of plugging cracks with sand in planning hydraulic fracturing operations. In this process the given and regulated parameters are the concentration of solid phase in the mixture and the rate of flow of liquid. Calculations of the same type can be used in other branches of technology involving the motion of suspensions, for example, in hydraulic conveyor systems, in canal building, etc.

§1. Analogy between the motion of a mixture of liquid and sand in a narrow channel and the percolation of immiscible liquids.

We will start by examining the mechanism by which a narrow flat horizontal channel (Fig. 1) is filled with sand suspended in a viscous liquid. In [4] it was shown that the mechanism of the motion of sand in a horizontal crack can be described in terms of the layered motion of two liquids of different viscosity. This possibility is realized if it is assumed that the viscosity of the principal carrier liquid is equal to μ_0, while the viscosity of the mixture of liquid and sand μ_1 is given by the relation

$$\mu_1 = \mu_0 e^{3.18c} . \tag{1.1}$$

Here c is the volume concentration of solid phase in the mixture.

Experiments [4] have confirmed the correctness of using the above-mentioned scheme and, moreover, show that, except for cases where the mixture pumped into the crack is very lean in sand, the concentration of solid phase in the lower layer is always about 0.5. The same reference [4] gives an analytic solution of the problem of motion of two liquids with different viscosities in a straight narrow channel with parallel impermeable walls.

Using the analogy between the two-layer motion of viscous liquids and the motion of a mixture of liquid and sand in a narrow channel, one can write expressions for the velocities of the liquid v_0 and the liquid-sand mixture v_1 referred to the full width of the channel w:

$$v_0 = \frac{w^2}{12\mu_0}\frac{\Delta p_c}{l} k_0(\sigma, \varepsilon), \qquad v_1 = \frac{w^2}{12\mu_1}\frac{\Delta p_c}{l} k_1(\sigma, \varepsilon),$$
$$\sigma = e^{3.18\varphi}, \qquad \varepsilon = \frac{h}{w} . \tag{1.2}$$

Here Δp_c is the pressure drop along the length l of the channel, and φ is the volume concentration of sand in the lower layer.

Actually, the liquid in question moves in the upper layer, but its rate of flow at any point is related to the entire cross section. Using the results of [4], we obtain expressions for the "relative conductivities" of the liquid and the mixture in the channel [7].

$$k_0 = \frac{2(2-3\varepsilon+\varepsilon^3)(\sigma-\varepsilon\sigma+\varepsilon)+3(\varepsilon^2+\sigma-\varepsilon^2\sigma)(2\varepsilon-\varepsilon^2-1)}{(1-\varepsilon)\sigma+\varepsilon},$$
$$k_1 = \varepsilon^2\frac{\varepsilon^2-\sigma(\varepsilon^2+2\varepsilon-3)}{\sigma(1-\varepsilon)+\varepsilon}. \tag{1.3}$$

Replacing $\Delta p_c/l$ in (1.2) by the differential form, we get

$$v_0 = -\frac{w^2}{12\mu_0}k_0(\sigma, \varepsilon)\frac{\partial p}{\partial x}, \qquad v_1 = -\frac{w^2}{12\mu_1}k_1(\sigma, \varepsilon)\frac{\partial p}{\partial x}. \tag{1.4}$$

It is easy to see that equations (1.4) are analogous to the law of percolation of immiscible liquids (for example, oil and water) in a porous medium. The quantities $k_0(\sigma, \varepsilon)$, $k_1(\sigma, \varepsilon)$ entering into these expressions are analogs of the relative permeabilities; $\varepsilon = hw^{-1}$, the relative thickness of the layer of sand moving in the channel, is the analog of the saturation, and σ is an auxiliary parameter taking into account the apparent viscosity of the mixture.

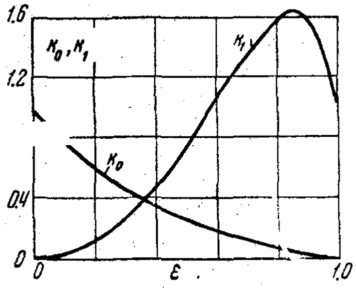

Fig. 2

If, in accordance with [4], it is assumed that in the lower layer $\varphi \approx 0.5$, then $\sigma = e^{3.18 \cdot 0.5} \approx 4.9$. The relations $k_0 = k_0(\sigma, \varepsilon)$ and $k_1(\sigma, \varepsilon)$, computed for $\sigma = 4.9$, are shown in Fig. 2. These relations differ from the curves of relative permeability for porous media in that at specific values of $\varepsilon < 1$ the relative permeability for the mixture K_1 is greater than at $\varepsilon = 1$. This behavior of the curve $k_1 = k_1(\sigma, \varepsilon)$ has an explanation. Consider, for example, the velocity profiles in the channel constructed for $\varepsilon = 0.8$ and $\varepsilon = 1$ (Fig. 3) in the form of the relation

$$\varepsilon = \varepsilon(S), \qquad S = \frac{2v_x \mu_0 \sigma}{w \Delta p_c / l}.$$

Here v_x is the true velocity in the direction of the x axes (Fig. 1). Although at $\varepsilon = 1$ the mixture completely fills the channel, i.e., the cross section of the channel occupied by liquid-sand mixture is in this case greater than for $\varepsilon = 0.8$, the flow rate of mixture will nonetheless be smaller than for $\varepsilon = 0.8$: at the top, instead of a fixed upper wall, there is a layer of moving liquid; therefore the area of the flow velocity diagram for the mixture at $\varepsilon = 0.8$ will be greater than at $\varepsilon = 1$ (Fig. 3). This effect is analogous to the so-called "lubricating effect" in pipe flow.

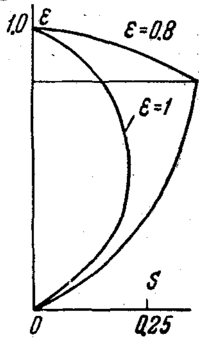

Fig. 3

The analogy between the motion of sand in horizontal channels and the percolation of a non-uniform liquid [7] makes it possible to use the Bukley-Leverett theory [8-10] to calculate the filling of a channel with sand. However, the following fact should be kept in mind. The scheme used above is suitable for describing the motion of a mixture of liquid and sand only if the channel is sufficiently narrow. If, however, the crack is a wide one (width greater than 10 diameters of the solid phase particles), then a "dead zone" consisting of settled motionless sand forms in the bottom of the channel. The description of this case requires a different scheme.

§ 2. Sanding up of straight flat narrow channels

First, we derive the equation of continuity of flow of a liquid and a liquid-sand mixture in a channel. We assume that the width of the channel $w = w(x)$ varies only slightly with variation of the coordinate. Then, considering the balance for a mixture in a channel of variable cross section, we get the following continuity equation for the mixture:

$$v_1 \frac{\partial w}{\partial x} + w \frac{\partial v_1}{\partial x} + w \frac{\partial \varepsilon}{\partial t} = 0; \tag{2.1}$$

and for the pure liquid

$$v_0 \frac{\partial w}{\partial x} + w \frac{\partial v_0}{\partial x} - w \frac{\partial \varepsilon}{\partial t} = 0 \quad (t \text{ — time}). \tag{2.2}$$

Grouping terms in (2.1), (2.2) and adding, we get $w(v_0 + v_1) = \text{const}$. Substituting (1.4) into (2.1) and (2.2), we obtain the equations of motion of the mixture in the channel. Further, following the Bukley-Leverett method [8-10], we can obtain the relation

$$F(\sigma, \varepsilon) = \frac{k_1}{k_1 + (\mu_1/\mu_0) k_0}. \tag{2.3}$$

The graph of this function at $\sigma = 4.9$ is presented in Fig. 4. We have

$$-Fq = wv_1, \qquad q = -w(v_1 + v_0) = \text{const}. \tag{2.4}$$

Substituting (2.4) into (2.1), we get

$$\frac{\partial F}{\partial \varepsilon} q \frac{\partial \varepsilon}{\partial x} + w \frac{\partial \varepsilon}{\partial t} = 0. \tag{2.5}$$

The solution of equation (2.5) for w = const has the form [8-10]

$$x = F'qt + \text{const}, \qquad F = dF / d\varepsilon. \tag{2.6}$$

If it is assumed that all the ε are equal to zero at t = 0, then

$$x = F'qt, \qquad \text{or} \qquad \xi = x / qt = F'(\varepsilon). \tag{2.7}$$

As follows from (2.7), the relative height of the layer of sand ε varies along the coordinate x or the dimensionless coordinate ξ. Let the dimensionless coordinate of the leading edge (front) of the sand be equal to ξ_*. Then, on the basis of the balance for the sand pumped into the channel, we can write

$$v = qt = \int_0^{x_*} w\varepsilon\, dx \quad \text{or} \quad \int_0^{\xi_*} \varepsilon(\xi)\, d\xi = 1, \tag{2.8}$$

It can be shown that, as in the case of motion of immiscible liquids [8-10], there is a relation for determining ξ_*

$$F'(\varepsilon_*) = F(\varepsilon_*)\varepsilon_*^{-1}. \tag{2.9}$$

The point ξ_* is the point of tangency of the straight line [8] drawn from the coordinate origin to the curve $F = F(\xi)$.

However, in the case of propagation of sand in a channel the boundary conditions may be different from those in [8]. In the given case at the boundary $x = 0$ we are given the rate of flow of the mixture of liquid and sand and the rate

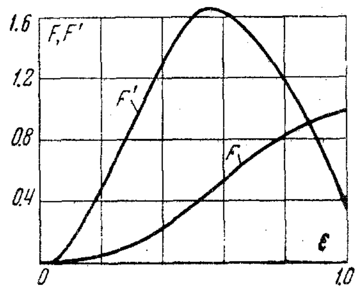

Fig. 4

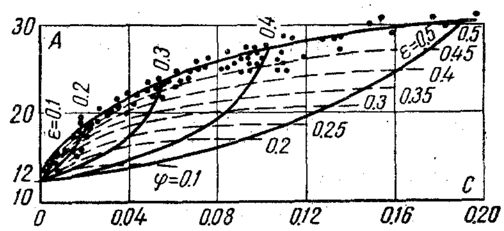

Fig. 5

of flow of the pure liquid Q or, in other words, the total rate of flow of the liquid and the concentration of sand c in the liquid. With this condition, in accordance with the balance, we have

$$v_1\varphi = Qc. \tag{2.10}$$

We now find the quantity v_0. It was remarked above that pure liquid flows in the upper layer of the channel, and in the lower layer a suspension of approximately constant concentration $\varphi \approx 0.5$. However, the relative velocities of the sand particles and the liquid in the lower layer are different, since the liquid moves through a "sand grating." Theoretically, it is difficult to determine how much liquid passes through the sand in unit time. However, there are experimental data [4] according to which, for a rectangular channel, we have

$$A(c) = \frac{\Delta p_c}{l}\frac{w^3}{\mu_0 Q}. \tag{2.11}$$

The relation $A = A(c)$ is shown in Fig. 5. In differential form this relation may be written

$$A(c) = -\frac{dp}{\partial x}\frac{w^3}{\mu_0 Q}, \quad \text{or} \quad -\frac{\partial p}{\partial x} = \frac{A(c)\mu_0 Q}{w^3}. \tag{2.12}$$

At the same time[1]

$$(v_0)_b = -\frac{w^3}{12\mu_0}\left(\frac{\partial p}{\partial x}\right)_b k_0(\varphi, \varepsilon_b), \qquad (v_1)_b = -\left(\frac{w^3}{12\mu_1}\right)_b k_1(\varphi, \varepsilon_b). \tag{2.13}$$

Hence, using (2.11), we get

$$(v_0)_b = {}^1/_{12}A(c)Qk_0(\varphi, \varepsilon_b). \tag{2.14}$$

In (2.14) v_0 and ε_b are unknown. In order to find them, we use relation (2.13) for the mixture. From (2.13) and (2.10) for $\varphi = 0.5$ we get

$$v_1 = \frac{A(c)\mu_0}{12\mu_1}k_1(\varphi, \varepsilon_b) \approx 2c. \tag{2.15}$$

Taking into account (1.1), we have

$$A(c)k_1(\varphi, \varepsilon_b) = 24ce^{3.18\varphi}. \tag{2.16}$$

[1]The subscript b denotes the corresponding quantities at the boundary of the channel.

Hence, knowing c and putting $\varphi \approx 0.5$, we determine ε_b. Then, knowing ε_b, we compute $(v_0)_b$ from (2.14).

Thus, we are given: $(v_0)_b$ = const, $(v_1)_b$ = const, and $\varphi = 0.5$. Hence, in accordance with (2.3) ε_b = const. In this case, ε_b may take any value, as distinct from the Bukley-Leverett problem [8], where we immediately have $\rho_b = 1$ for $t \gg 0$.

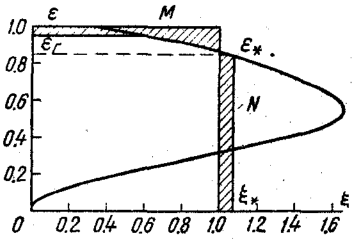

Fig. 6

The function $F = F(\varepsilon)$ reflects the fraction of the flow velocity of the sand-liquid mixture in the total flow along the channel. The typical form of the function $F(\varepsilon)$ at $\sigma = 4.9$ is shown in Fig. 4, which also gives the relation $F' = F'(\varepsilon)$. If we construct the graph of this relation, after changing coordinates, then, in accordance with (2.8), the position of ξ_* on the graph of Fig. 6 will be determined from the condition that the area defined by the curve of distribution of the layer of sand-liquid mixture is equal to unity, i.e., the shaded areas M and N in Fig. 6 must be equal. The example shown in Fig. 6 is for the case $\varepsilon_b = 0.95$.

Using the proposed method we made calculations for a straight flat channel filled with sand at different C and Q for cases when the width of the channel w is equal to 0.8 and 0.3 cm. The results of the calculations showed that the distribution of the layer of sand, in accordance with the Bukley-Leverett theory, proceeds at $\varepsilon \geq 0.76$, while at smaller values of ε the layer of sand has practically the same height over the entire length of the channel.

§ 3. Sanding up of a radial horizontal crack

Using the same reasoning as that given in § 2, we obtain the continuity equation for a radial horizontal channel of variable cross section. The equations for the two phases have the form

$$w\frac{\partial\varepsilon}{\partial t}+\frac{v_1 w}{r}+w\frac{\partial v_1}{\partial r}+v_1\frac{\partial w}{\partial r}=0, \tag{3.1}$$

$$-w\frac{\partial\varepsilon}{\partial t}+\frac{v_0 w}{r}+w\frac{\partial v_0}{\partial r}+v_0\frac{\partial w}{\partial r}=0. \tag{3.2}$$

For w = const we obtain from these equations relations analogous to the Bukley-Leverett equations.

Expressions (3.1) and (3.2) can be written in the form

$$w\frac{\partial\varepsilon}{\partial t}+\frac{v_1 w}{r}+\frac{\partial(v_1 w)}{\partial r}=0,\qquad -w\frac{\partial\varepsilon}{\partial t}+\frac{v_0 w}{r}+\frac{\partial(v_0 w)}{\partial r}=0. \tag{3.3}$$

Adding the last two equations, we get

$$\frac{\partial}{\partial r}[(v_1+v_0)wr]=0,\quad\text{or}\quad (v_2+v_0)wr=\text{const}. \tag{3.4}$$

Further, in accordance with the Bukley-Leverett method, by analogy with § 2 we get

$$F=\frac{v_1 rw}{(v_1+v_0)rw}. \tag{3.5}$$

Since $(v_1+v_0)rw = \text{const} = q$,

$$\frac{1}{r}\frac{\partial}{\partial r}(v_1 wr)=\frac{1}{r}\frac{\partial}{\partial r}(Fq),\quad\text{or}\quad w\frac{\partial\varepsilon}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(Fq)=0. \tag{3.6}$$

Equation (3.6) for ε = const assumes the form

$$\frac{dr}{dt}=\frac{F'q}{wr}. \tag{3.7}$$

The solution of this equation is

$$\lambda(R)-\lambda(r_c)=F'\quad\left(\lambda=\frac{1}{v}\int_{r_c}^{R}wrdr\right). \tag{3.8}$$

Here R is the variable radius, r_c is the radius of the well (pipe), through which the suspension is pumped into the radial crack. Now let us consider the expression

$$\int_{r_c}^{r_*}2\pi rw\varepsilon\,dr=v,$$

where r_* is the coordinate of the leading edge (front) of the sand in the channel.

From (3.8) we have $v^{-1}wrdr = d\lambda$ and, consequently, we can write

$$\int_{\lambda(r_c)}^{\lambda(r_*)} 2\pi\varepsilon(\lambda)\, d\lambda = 1. \qquad (3.9)$$

Since $\lambda(r_c) = \mathrm{const}$, from (3.8) we have

$$d\xi = d[\lambda(R) - \lambda(r_c)], \quad \text{or} \quad 2\pi \int_0^{\xi(r_*)} \varepsilon(\xi)\, d\xi = 1, \qquad (3.10)$$

Thus, for the case of a radial crack we get a relation analogous to the relation for a straight channel, but the area defined by the curve of distribution of the layer of sand-liquid mixture in the channel must be equal to r/2.

REFERENCES

1. Yu. P. Zheltov, Hydraulic Fracturing of Formations [in Russian], Gostoptekhizdat, Moscow, 1957.

2. G. K. Maksimovich, Hydraulic Fracturing of Oil-Bearing Formations [in Russian], Gostoptekhizdat, Moscow, 1957.

3. Theory and Practice of Hydraulic Fracturing of Formations [in Russian], Proc. of All-Union Conference, Gostoptekhizdat, 1957.

4. V. S. Dorozhkin, Yu. V. Zheltov, and Yu. P. Zheltov, "Motion of liquid-sand mixtures in wells and cracks," Izv. AN SSSR, OTN, no. 11, 1958.

5. Yu. N. Vasil'ev, "Motion of sand-liquid mixtures along channels," Neft. kh-vo, no. 10, 1958.

6. N. N. Shan'gin and A. M. Izyumova, "Motion of sand in a horizontal crack formed during hydraulic fracturing of formations," Neft. kh-vo, no. 4, 1958.

7. Yu. P. Zheltov, Studies of the Processes of Deformation and Percolation in the Exploitation of Oil-Bearing Formations [in Russian], Author's abstract of doctoral dissertation. All-Union Scientific Research Institute of Petroleum and Gas, Moscow, 1962.

8. J. Bukley and M. Leverett, "Mechanism of fluid displacement in sands," Trans. AIME, vol. 146, 1942.

9. D. A. Efros, "Determination of relative permeabilities and distribution functions in connection with the displacement of oil by water,"DAN SSSR, vol. 100, no. 5.

10. V. M. Ryzhik, "Review of research on the mutual displacement of immiscible liquids from a porous medium," Izv. AN SSSR, OTN, Mekhanika i mashinostroenie, no. 2, 1961.

18 June 1964 Moscow